2019.10.20
JA1IWP

How to make a loading coil (How to estimate inductance)

1. Relationship between coil shape, number of turns and inductance

Various homepages show how to calculate the inductance of an air-core coil using the Nagaoka coefficient. It seems that the Nagaoka coefficient can be calculated with an elliptic function, but there are already numerical tables as calculation results (for example, HP of Nippon Universal Electric Co., Ltd.), and if you use them, you can easily predict the inductance of the coil you are going to make. be able to.

Here, a program for calculating the inductance of the air-core coil by Scilab and a program for obtaining the (polynomial) coefficient when the Nagaoka coefficient is approximated by a quadratic polynomial are shown by the least squares method.

$$L = A \times (2\pi)^2 \times \mu s \times a^2 \times N^2 / b \times 10^{-7} \ [H]$$

L: Inductance [H]
A : Nagaoka coefficient A(2a/b) [-]
μs: Permeability(=1 : Air core)
a : Coil radius [m]
N : Number of turns [times]
b : Coil winding width [m]

$$L = A \times (2\pi)^2 \times \mu s \times a^2 \times N^2 / b \times 10^{-4} \ [\mu H]$$

L: Inductance [μH]
A : Nagaoka coefficient A(2a/b) [-]
μs: Permeability(=1 : 空芯)
a : Coil radius [mm]
N : Number of turns [回]
b : Coil winding width [mm]

2. Nagaoka coefficient

(1) Nagaoka coefficient table

2a/b	N. coef. A	2a/b	N. coef. A	2a/b	N. coef. A	2a/b	N. coef. A
0.00	1.000	0.55	0.803	1.10	0.667	2.50	0.472
0.05	0.979	0.60	0.789	1.20	0.648	3.00	0.429
0.1	0.959	0.65	0.775	1.30	0.629	3.50	0.394
0.15	0.939	0.70	0.761	1.40	0.611	4.00	0.365
0.20	0.920	0.75	0.748	1.50	0.595	4.50	0.341
0.25	0.902	0.80	0.735	1.60	0.580	5.00	0.320
0.30	0.884	0.85	0.723	1.70	0.565	6.00	0.285
0.35	0.867	0.90	0.711	1.80	0.551	7.00	0.258
0.40	0.850	0.95	0.700	1.90	0.538	8.00	0.237
0.45	0.834	1.00	0.688	2.00	0.526	9.00	0.219
0.50	0.818					10.00	0.203

(from Nihon Universal Electric Co., Ltd HP)

(2) Second-order polynomial approximation of Nagaoka coefficient

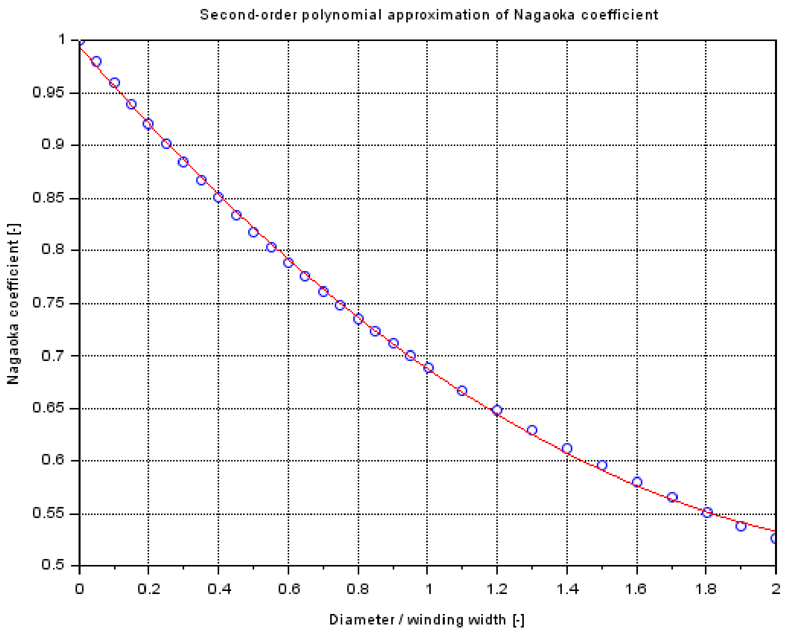
The Nagaoka coefficient is approximated by a 2a / b quadratic polynomial, and the coefficient is calculated by the least squares method. (Refer to the appendix for details) (Matlab can easily calculate with an M-file called "polyfit"!)

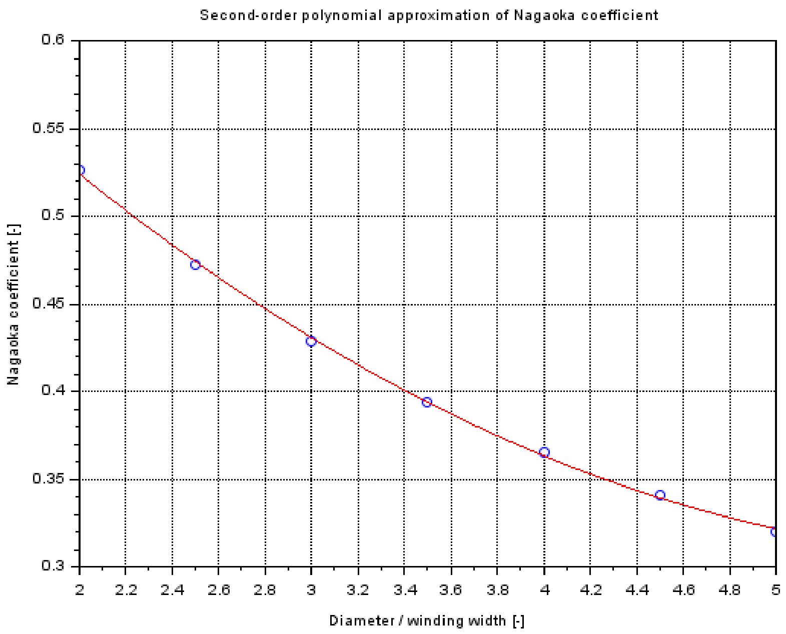
(1) 2a / b (diameter / winding width) = 0.0 to 2.0

$$A = 0.0761 \times (2a / b)^2 - 0.383 \times (2a / b) + 0.994$$

(2) 2a / b (diameter / winding width) = 2.0 to 5.0

$$A = 0.0130 \times (2a / b)^2 - 0.158 \times (2a / b) + 0.788$$





3. Loading coil trial result

Various coils were prototyped, and the calculated value (Lc) using the Nagaoka coefficient and the measured value (Lm) using the LCR meter (Sanwa LCR700, @ 100kHz) were compared.

In conclusion, the coil inductance calculation using the Nagaoka coefficient is in good agreement with the actual hardware.

No.	Bobbin Material	2a (dia.) [mm]	B (Winding width) [mm]	N (Number of turns)[times]	2a/b [-]	Winding dia. [mm]	N coef. A[-]	Lc (calc.) [μH]	Lm (act.) [μH]	(Lc-Lm) ／Lm[%]
1	paper※	30	36	42	0.833	0.8	0.727	31.7	31.2	1.6
2	Polycarbonate ＊	38	29	33	1.310	0.8	0.623	33.3	32.8	1.5
3	Polycarbonate	32	32	47	1.000	0.6	0.687	47.9	47.4	1.1
4	Polycarbonate	30	37	57	0.811	0.6	0.733	57.2	61.2	-6.5
5	Polycarbonate	26	32	46	0.813	0.6	0.733	32.3	32.0	1.0
6	VP pipe	26	49	55	0.531	0.8	0.812	33.5	34.1	-1.8
7	VP pipe	26	30	47	0.867	0.6	0.719	35.3	35.2	0.3
8	VP pipe	26	23	51	1.130	0.4	0.658	49.7	50.4	-1.4
9	VP pipe	18	20	45	0.900	0.4	0.711	23.0	23.5	-2.1
10	VP pipe	18	24	52	0.750	0.4	0.750	27.0	28.1	-3.9

※: Saran wrap core　＊: Cosmetic spray (purchased at 100 Yen shop)

Appendix 1. Air core coil inductance calculation program

```
// 空芯コイルのインダクタンス計算プログラム    2019.10.20
// 条件  比誘電率≒1.0
//        直径／巻き幅≦5.0 (5.0以上では0を返します)
// a:コイルの半径,b:コイルの巻き幅,n:コイルの巻き数 を変更し
// 「保存して実行」

clear

a = 15;       // [mm] コイルの半径
b = 36;       // [mm] コイルの巻き幅
n = 42;       // [回] コイルの巻き数

if 2*a/b > 2.0  then disp("error 2a/b > 5") end

// 長岡係数[A]の計算（２次多項式近似）
if 2*a/b <= 2.0
    then A = 0.0761*(2*a/b)^2-0.383*(2*a/b)+0.994;
    else A = 0.0130*(2*a/b)^2-0.158*(2*a/b)+0.788;
end

// インダクタンスの計算
L = A * (2*%pi)^2 * a^2 * n^2 / b * 1e-4   // [uH]

if 2*a/b > 5.0  then L = 0  end
disp(a,b,n,2*a/b,A,L)
if 2*a/b > 5.0  then disp("error 2a/b > 5") end
```

Appendix 2. Second-order polynomial approximation method (least squares method)

1. About least squares approximation

・basic way of thinking

Let the Nagaoka coefficient at the point of [X1 X2 ・ Xi ・ ・ Xn] be [Y1 Y2 ・ Yi ・ ・ Yn].
This is a quadratic function

Approximate with,

$$Y = a_2 X^2 + a_1 X + a_0$$

residual

$$e_i = Y_i - (a_2 X_i^2 + a_1 X_i + a_0) 。$$

Minimize the squared of the residual e^2、

$$e_i^2 = \{ Y_i - (a_2X_i^2 + a_1X_i + a_0) \}^2$$
$$= Y_i^2 - 2Y_i (a_2X_i^2 + a_1X_i + a_0) + (a_2X_i^2 + a_1X_i + a_0)^2$$
$$= Y_i^2 - 2Y_i a_2X_i^2 + 2Y_i a_1X_i + 2Y_i a_0 + a_2^2X_i^4 + 2a_2a_1X_i^3 + 2a_2a_0X_i^2 + a_1^2X_i^2 + 2a_1a_0X_i + a_0^2$$

Add up for all i = 1,2,・・,n、

$$\Sigma e_i^2 = \Sigma(Y_i^2 - 2Y_i a_2X_i^2 + 2Y_i a_1X_i + 2Y_i a_0 + a_2^2X_i^4 + 2a_2a_1X_i^3 + 2a_2a_0X_i^2 + a_1^2X_i^2 + 2a_1a_0X_i + a_0^2)$$
$$= \Sigma Y_i^2 - 2a_2\Sigma Y_iX_i^2 - 2a_1\Sigma Y_i X_i - 2a_0\Sigma Y_i + a_2^2\Sigma X_i^4 + 2a_2a_1\Sigma X_i^3 + 2a_2a_0\Sigma X_i^2 + a_1^2\Sigma X_i^2 - 2a_1a_0\Sigma X_i - a_0^2\Sigma(1)$$

The condition to make Σe_i^2 Minimize is,

$$\partial \Sigma e_i^2/\partial a_2 = \partial \Sigma e_i^2/\partial a_1 = \partial \Sigma e_i^2/\partial a_0 = 0$$

$$\partial \Sigma e_i^2/\partial a_2 = -2\Sigma Y_iX_i^2 + 2a_2\Sigma X_i^4 + 2a_1\Sigma X_i^3 + 2a_0\Sigma X_i^2 = 0$$
$$\partial \Sigma e_i^2/\partial a_1 = -2\Sigma Y_iX_i + 2a_2\Sigma X_i^3 + 2a_1\Sigma X_i^2 + 2a_0\Sigma X_i = 0$$
$$\partial \Sigma e_i^2/\partial a_0 = -2\Sigma Y_i + 2a_2\Sigma X_i^2 + 2a_1\Sigma X_i + 2a_0\Sigma(1) = 0$$

$$a_2\Sigma X_i^4 + a_1\Sigma X_i^3 + a_0\Sigma X_i^2 = \Sigma Y_iX_i^2$$
$$a_2\Sigma X_i^3 + a_1\Sigma X_i^2 + a_0\Sigma X_i = \Sigma Y_iX_i$$
$$a_2\Sigma X_i^2 + a_1\Sigma X_i + a_0\Sigma(1) = \Sigma Y_i$$

$$\begin{pmatrix} \Sigma X_i^4 & \Sigma X_i^3 & \Sigma X_i^2 \\ \Sigma X_i^3 & \Sigma X_i^2 & \Sigma X_i \\ \Sigma X_i^2 & \Sigma X_i & \Sigma(1) \end{pmatrix} \begin{pmatrix} a_2 \\ a_1 \\ a_0 \end{pmatrix} = \begin{pmatrix} \Sigma Y_iX_i^2 \\ \Sigma Y_iX_i \\ \Sigma Y_i \end{pmatrix}$$

$$\begin{pmatrix} a_2 \\ a_1 \\ a_0 \end{pmatrix} = \begin{pmatrix} \Sigma X_i^4 & \Sigma X_i^3 & \Sigma X_i^2 \\ \Sigma X_i^3 & \Sigma X_i^2 & \Sigma X_i \\ \Sigma X_i^2 & \Sigma X_i & \Sigma(1) \end{pmatrix}^{-1} \begin{pmatrix} \Sigma Y_iX_i^2 \\ \Sigma Y_iX_i \\ \Sigma Y_i \end{pmatrix}$$

$$Y = a_2 X^2 + a_1 X + a_0$$

·When approximating with 3rd order equation,

$$\begin{pmatrix} a3 \\ a2 \\ a1 \\ a0 \end{pmatrix} = \begin{pmatrix} \Sigma Xi^6 & \Sigma Xi^5 & \Sigma Xi^4 & \Sigma Xi^3 \\ \Sigma Xi^5 & \Sigma Xi^4 & \Sigma Xi^3 & \Sigma Xi^2 \\ \Sigma Xi^4 & \Sigma Xi^3 & \Sigma Xi^2 & \Sigma Xi \\ \Sigma Xi^3 & \Sigma Xi^2 & \Sigma Xi & \Sigma(1) \end{pmatrix}^{-1} \begin{pmatrix} \Sigma YiXi^3 \\ \Sigma YiXi^2 \\ \Sigma YiXi \\ \Sigma Yi \end{pmatrix}$$

$$Y = a3\, X^3 + a2\, X^2 + a1\, X + a0$$

Appendix 3. Second-order polynomial approximation of data by least squares method (Scilab program)

<< Diameter / Winding width = 0.0 to 2.0 >>

```
// 長岡係数の２次多項式近似（1）          2019.10.20
//    直径／巻き幅＝0.0〜2.0

clear
// X: コイル直径／巻き幅(0.0〜2.0)
X = [0.0 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60
0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00];
X = [X 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0]

// Y:  長岡係数
Y = [1.00 0.979 0.959 0.939 0.920 0.902 0.884 0.867 0.850 0.834 0.818
0.803 0.789 0.775 0.761 0.748 0.735 0.723 0.711 0.700 0.688];
Y = [Y 0.667 0.648 0.629 0.611 0.595 0.580 0.565 0.551 0.538 0.526]

sizez = size(X);

XX = X.*X; XXX= XX.*X; XXXX = XXX.*X; YX = Y.*X; YXX = YX.*X;
sXXXX=sum(XXXX); sXXX=sum(XXX); sXX=sum(XX); sX=sum(X);
sYXX=sum(YXX); sYX=sum(YX); sY=sum(Y);

D = [sXXXX sXXX sXX; sXXX sXX sX; sXX sX sizez(2)]
E = [sYXX sYX sY]'
F = inv(D)*E
a2 = F(1); a1 = F(2); a0 = F(3)

// ２次多項式近似のデータ作成
x = 0:0.01:2;
y = a2*x.^2 + a1*x + a0;

// グラフ描画
plot(X,Y,'o')            // データプロット
plot(x,y,'r')            // ２次式近似描画
xgrid(), zoom_rect([0 0.5 2 1])
xtitle('長岡係数の２次多項式近似','直径／巻き幅[-]','長岡係数[-]')
```

<< Diameter / Winding width = 2.0 to 5.0 >>

```
// 長岡係数の２次多項式近似（2）          2019.10.20
//    直径／巻き幅=2.0～5.0

clear
// X: コイル直径／巻き幅(2.0～5.0)
X = [2 2.5 3 3.5 4 4.5 5];

// Y:  長岡係数
Y = [0.526 0.472 0.429 0.394 0.365 0.341 0.320];

sizez = size(X);

XX = X.*X; XXX= XX.*X; XXXX = XXX.*X; YX = Y.*X; YXX = YX.*X;
sXXXX=sum(XXXX); sXXX=sum(XXX); sXX=sum(XX); sX=sum(X);
sYXX=sum(YXX); sYX=sum(YX); sY=sum(Y);

D = [sXXXX sXXX sXX; sXXX sXX sX; sXX sX sizez(2)]
E = [sYXX sYX sY]'
F = inv(D)*E
a2 = F(1); a1 = F(2); a0 = F(3)

// ２次多項式近似式のデータ作成
x = 2:0.01:5;
y = a2*x.^2 + a1*x + a0;

// グラフ描画
plot(X,Y,'o')              // データプロット
plot(x,y,'r')              // ２次式近似描画
xgrid(), zoom_rect([2 0.3 5 0.6])
xtitle('長岡係数の２次多項式近似','直径／巻き幅[-]','長岡係数[-]')
```